

Dulong.....	D .....	1015.97	miles.
Hess.....	H .....	1017.40	“
Crawford.....	C .....	1052.73	“
Grassi.....	G .....	1013.72	“
Favre and Silbermann...	F .....	1013.60	“
Adams.....	A .....	988.63	“
Mean of D, H, G, F.....		1015.18	“
General Mean.....		1017.01	“

If we assume the correctness of the general mean,  $d = \delta \times \sqrt{\frac{\pi}{8}} =$

$$1129.61; \tau^{\delta} = \tau_1^{\delta} \times \sqrt{\frac{g^{\delta}}{g}} = 578.5 \times \frac{1129.61}{7925.64} = 82.45 \text{ seconds}; x = d \div$$

$$4 = 282.4; T^d = \tau^{\delta} \div \left[ \frac{\delta}{d} \right]^{\frac{3}{2}} = 96.515 \text{ seconds}; v = 2 \pi x \div T^d =$$

$$18.3844 \text{ miles}; \gamma = v \times 1 \text{ year (in seconds)} \div 2 \pi = 92,338,000 \text{ miles};$$

$$\mu = \gamma \div x = 326,980.$$

This approximation is subject to correction for possible imperfect elasticity of hydrogen, æthereal resistance, and orbital eccentricity. From various considerations I am inclined to believe that the aggregate corrections for the value of  $\gamma$ , cannot exceed to one and a-half per cent. of the above amount.

#### NOTE ON PLANETO-TAXIS.

BY PLINY EARLE CHASE.

(*Read before the American Philosophical Society, March 7th, 1873.*)

I am not aware that any reason has ever been assigned for the planetary harmony which is formulated in "Bode's Law," or that any attempt has been made to show that the failure of the analogy, in the case of Neptune, is really only one of those apparent exceptions which serve to establish general rules on a firmer basis.

The many evidences which I have already adduced, of simple relationships between planetary positions and centres of oscillation, seem to furnish the needed data for verifying the law, as a simple and natural resultant of equilibrating forces, and not a mere accidental coincidence. If a nebulous mass were set in rotation, each of its equatorial radii might be regarded as a simple pendulum, with a tendency to vibrate in the same time as its centre of oscillation, which tendency might be expected to produce an aggregation at that centre.

If we start from the circularly divided radius next within the orbit of Mercury, ( $\frac{\pi}{32} r = .0982$ ), and add multiples of the next following radius ( $\frac{\pi^2}{32} r = .3085$ ), we may form the first series (A) in the following table,

each term of which is within the limits of secular variation. The second and third series (B, C,) represent the mean perihelion and aphelion planetary distances. The fourth series (D) gives the mean distances; and the fifth (E) is derived from the first by simple systematic modifications.

## THEORETICAL AND OBSERVED PLANETARY POSITIONS.

		A	B	C	D	E
$\frac{\pi}{32}$		.098				
$\frac{\pi + \pi^2}{32}$	♃	.407	.319	.455	.387	.386
$\frac{\pi + 2\pi^2}{32}$	♀	.715	.698	.749	.723	.715
$\frac{\pi + 3\pi^2}{32}$	♁	1.024	.966	1.034	1.000	.998
$\frac{\pi + 5\pi^2}{32}$	♂	1.641	1.403	1.642	1.524	1.520
$\frac{\pi + 9\pi^2}{32}$	♄	2.875	2.201	3.420	2.810	
$\frac{\pi + 17\pi^2}{32}$	♅	5.342	4.978	5.427	5.203	5.209
$\frac{\pi + 33\pi^2}{32}$	♁	10.278	9.078	10.000	9.539	9.525
$\frac{\pi + 65\pi^2}{32}$	♁	20.150	18.322	20.043	19.183	19.153
$\frac{\pi + 97\pi^2}{32}$	♁	30.022	29.735	30.339	30.037	30.021

I have already spoken of corrections that seemed to be requisite in many of my analogies, on account of planetary eccentricities. If the A series be divided by a mean proportional between the average (major ÷ mean) radii vectores of Earth and Jupiter, the results will differ less from the actual planetary positions than those given by Bode's Law. The remarkably close approximations of the E series were obtained by using  $E^0$ ,  $E^{\frac{1}{3}}$ ,  $E^{\frac{2}{3}}$ ,  $E^1$  as divisors, ( $E = 1.079065 = \text{average } \frac{\text{major}}{\text{mean}} \text{ radius vector of Mars.}^*$ )

The planetary deviations are grouped in pairs, and also in exterior and interior systems. Neptune and Venus are not materially shifted; Mercury and Uranus are divided by  $E^{\frac{2}{3}}$ ; Earth and Jupiter by  $E^{\frac{1}{3}}$ ; Mars and Saturn by  $E^1$ . The ratios of the divisors to radius vector, time, and velocity, may have important bearings.

\* The values of the mean planetary eccentricities were taken from Stockwell's recent paper on "Secular Variations of the Elements of the Orbits of the Eight Principal Planets."<sup>11</sup>

The theoretical distance of Neptune (in column A) appears to be an *exact* mean proportional between Mercury's theoretical distance and the modulus of light. That modulus, according to this determination, is  $476,198 \times$  Sun's radius; according to Struve's value of the constant of aberration, it is  $(475,969.23 \pm 258.45) \times$  solar radius.

The theoretical series is symmetrical, in having three terms in arithmetical progression at either extremity. This analogy is more nearly carried out in the actual positions of the three exterior planets, which have been regarded as exceptional, than in those of the three interior planets, which have been considered normal.

The theoretical positions of Mercury and Venus, are at centres of direct and reverse oscillation between Earth and  $\frac{\pi}{32}$ ; those of Uranus and Saturn, at similar centres between Sun and Neptune.

The successive doubling of the differences, places each of the theoretical intermediate planets at a centre of oscillation between the next inferior and the next superior planet.

The deviations from theoretical positions, in consequence of mutual planetary disturbances, distribute the planets in various symmetrical ways.

The exponents of the divisor, E, are arranged symmetrically in pairs. ( $\frac{2}{3}, 0; \frac{1}{3}, 1; \frac{1}{3}, 1; \frac{2}{3}, 0$ .)

The four central planets are grouped, by their divisors, in alternate pairs; Earth, Jupiter; Mars, Saturn.

The four terminal planets are similarly grouped; Neptune, Venus; Uranus, Mercury.

If the division by  $\pi$  be thrice repeated, below the theoretical position of Mercury, we obtain, very nearly,  $(\pi-1) \times$  solar radius, or the diameter of the circle described by the centre of gravity of Sun and Jupiter.

## ROTATION OF THE SUN AND THE INTRA-ASTEROIDAL PLANETS.

BY PLINY EARLE CHASE.

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The well known tendency to synchronism in concurrent vibrations, has encouraged me to look for some significant harmony between the lengths of solar and planetary days and times of fall to the centre of the system.

The middle term in my series of alternate planetary distances, differs from the others in having a multiple significance, representing, as it does, a mean position in the asteroidal belt and the orbital major axis of Mars. It has also simple relationships to the distances and rotation-times of the intra-asteroidal planets, which serve to connect the diurnal with the annual motions, and both with the equilibrating forces of the Sun.

Since the velocities of falling and oscillating bodies are proportioned